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Mathematical modelling of accidental gas releases

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Abstract

The increasing use of natural gas entails large networks, located mostly in highly populated zones. Therefore, any accidental releases of gas that might occur through damaged pipes imply a risk which must be controlled. In such cases, the prediction of release flow-rate and time of duration of the emergency is very important. However, the mathematical models currently available for performing this prediction show a major gap in the range of conditions over which they can be applied with a reasonable degree of accuracy. Furthermore, they do not take into consideration the unsteady state existing when a safety device is closed after a certain time of release. In this paper, a new model is developed as a combination of the classical 'hole' and 'pipe' models, for the calculation of gas releases in distribution systems at medium and low pressures. This model can be applied to these two cases (small hole in a tank or a pipe and full bore holes)

Abbreviations: A', Parameter defined by Eq. (47); A_c, Area of the cross section of the pipe (m^2); A_{or}, Hole area (m^2); B, Parameter defined by Eq. (47); C, Velocity of the sound (m/s); for ideal gases $C=(k \cdot M/(R \cdot T))^{1/2}$; C', Parameter defined by Eq. (47); CPR, critical pressure ratio (-); D, Pipe diameter (m or mm); d_{or}, Hole diameter (mm); F, Function defined by Eq. (45) (-); f, Friction factor (-); G, Mass flux (kg/m^2 s); k, Heat capacity ratio (-); K_i, Pressure drop coefficient for each fitting (-); L, Pipe length (m); L_e, Equivalent pipe length (m); M, Molecular weight (kg/kmol); m, Mass of gas contained in the pipe (kg); Ma, Mach number, u/C (-); N_i, number of fittings (-); P, Pressure (Pa or bar abs); P_c, critical pressure (Pa or bar abs); Q_m, Mass discharge rate (kg/s or Nm^3/h); R, Ideal gas constant (J/kmol K); Re, Reynolds number, $Du\rho/\mu$ (-); T, Temperature (K); t, Time (s); t_c, Critical time (s); u, Velocity of the gas (m/s); V_P, Pipe volume (m^3); Y, Parameter defined by Eq. (14) (-); *Greek letters:* α, Parameter defined by Eq. (36); β, Parameter defined by Eq. (39); Φ, Parameter defined by Eq. (47); ρ, Density (kg/m^3); μ, Viscosity (kg/m s); *Subscripts:* 0, Steady-state; 0_{sub}, Steady state or initial values in subsonic flow; 1, At the beginning of the pipe; 2, Inside the pipe and on a level with the hole; 3, At the hole; a, In the surroundings, at atmospheric pressure; E, Regulator inlet conditions; i, Initial conditions; j, Any point

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and also to the intermediate situation, a large hole in a pipe. Its application to various accident scenarios is discussed.

Keywords: Mathematical modelling; Gas releases; Accidental

1. Introduction

The consumption of natural gas in the industrialized countries has increased steadily in recent years. Its clean-combustion characteristics and its ease of distribution at low pressures justify its wide use (both for the generation of electricity and as a domestic fuel), which has doubled from 1990 to 1994, resulting in a worldwide consumption of 2170×10^{12} m^3 in 1994.

This use has meant installing and maintaining complex piping systems to transport and distribute gas, a great deal of them located in highly populated zones. Due to these facts, accidents caused by the loss of containment of natural gas can involve substantial economic losses and even victims amongst the population.

Polyethylene tubing has largely replaced the steel or cast iron gas pipes which were used traditionally. Polyethylene pipe has several advantages: it is easier to install, has good mechanical properties and it is not corroded by damp soils. Thousands of kilometers of this pipe have been installed and its use is still increasing.

Gas pipes, which are mostly installed underground, can be damaged by various activities: underground work, bull-dozers, etc. If there is a partial or complete breaking of the pipe, the result will always be a loss of containment. Such a situation can be no more than an incident if the leak is small or is rapidly detected, or can lead to an accident depending on the circumstances.

A recent survey of 185 accidents involving natural gas [1] showed that, of the total, 131 were caused during transportation, either by road, railway, ship or pipeline. The analysis of these data clearly shows the relatively high frequency of accidents in pipes: 127 of them occurred in piping systems. The most frequent causes of the accidents were mechanical failure, impact failure, human error and external events. Amongst the accidents arising from impact failure (39), the most frequent specific cause was excavating machinery (21 accidents), followed by vehicles (5) and heavy objects (5). Other specific causes due to external events were ground subsidence (4 cases) and sabotage/vandalism (4 cases).

In the analysis of such accidents, or when forecasting the consequences of this type of hypothetical situation, the first stage is to estimate the flow-rate at which the gas is being released through the damaged pipe [2]. This is a complex problem, which in fact begins with the definition of the 'hole diameter': a particular opening size must be assumed; it can be a small hole, a large one or even the whole cross-section of the pipe if it is completely broken. Then, the flow-rate through this hole must be calculated, both for the case of constant flow-rate and for the case of decreasing velocity (if an emergency valve is automatically closed, for example). In this paper, a mathematical model is described which allows the prediction of the flow-rate of gas through a damaged pipe.

2. Accidental release

In the flow of compressible fluids there can be significant changes in fluid density; this implies significant variation as well of pressure and temperature. Therefore, the analysis of such systems involves four equations: the equation of state and those of continuity, momentum and energy. This makes the analysis rather complicated. To simplify it, it is usually assumed that the flow is reversible and adiabatic, these conditions implying isentropic flow. Furthermore, it is often assumed that the fluid is a perfect gas with constant specific heat (average value).

In the case of gas release the existing models describe two situations: (a) gas flow through a hole, the pipe being considered to be like a tank: usually called 'hole models'; (b) gas flow through a hole which corresponds to the complete breaking of the pipe: 'pipe models'.

Hole models have been widely treated in the literature. Woodward and Mudan [3] also developed a model for the calculation of liquid and gas discharge rates through holes in process vessels, which takes into account the decrease in pressure as a function of time. Levenspiel [4] developed an interesting model which was later adapted to the estimation of accidental releases by Crowl and Louvar [5]. It considers the release of a gas contained in a tank through a hole; two assumptions are made: pressure inside the tank is constant and gas expansion is isentropic.

This model can be applied to the loss of containment of a gas from a pipe if the hole is small; however, for large holes the model overpredicts the flow-rate. The pressure inside the pipe is considered to be constant, and therefore unsteady state is not taken into account. Neither does the model take into account either changes in gas exit velocity (thus implying constant outlet temperature) or pressure drop in the pipe. All these aspects make this model adequate for the prediction of release through a hole in a tank, but not for the description of release through a hole in a pipe.

The same authors [4,5] also describe a model for the case of complete rupture of the pipe, assuming adiabatic flow (i.e. without any heat loss or heating from the surroundings). This model assumes isentropic release; a constant pressure is assumed at an initial point in the pipe, and pressure drop along it is taken into account. The model gives good predictions for the case of complete breaking of the pipe, but it cannot be applied to the flow through holes with a diameter smaller than the pipe diameter. Numerical solutions have been treated by Györi [6], Cochran [7] and Farina [8]. Giraldo [9] applied the pipe model to a non-adiabatic case considering heat transfer between the pipe and the surroundings.

These two models leave a major gap: all the cases ranging from a relatively small hole up to a large orifice with a diameter smaller than that of the pipe (hole models can be applied to small orifices, and pipe models to the complete breaking of the pipe). Nevertheless, this range of hole diameters corresponds to a situation which can be found in those accidents caused by accidental releases from pipes. Furthermore, the aforementioned models do not take into account the unsteady-state regime found, for example, when a safety device is closed and pressure and gas flow-rate decrease progressively. Although the hole models and pipe models are often sufficient for hazard analysis (case of an hypothetical incident), a more accurate model can be useful, especially for incident investigation purposes (when more precise information on the release is available).

A new model bridging this gap would therefore be very useful for the case of accidental gas releases. In the following sections a model is described which takes into account all these aspects, thus covering all the possible hole diameters—assuming diverse hypothesis described in Section 3—for the release of gas from a damaged pipe. This new model, which is a combination of the two aforementioned models, should lead to the same predictions as the hole models for the case of small holes (as, in this case, pressure inside the pipe will undergo little variation); and should likewise give the same prediction as the pipe model for those cases in which the hole diameter is equal to the pipe diameter.

This model has been developed for gas distribution systems operating at medium and low pressures (i.e. pressures at which ideal gas behaviour exists). It is therefore different from other models intended for dealing with accidental releases occurring in long, high-pressure gas pipelines [10,11], in which the propagation of pressure disturbances within the pipe should be also taken into account.

3. Hole–pipe generalized model

The system analyzed here is shown schematically in Fig. 1. From the distribution main, there is a length of pipe (L) after which there is a hole with a certain diameter through which the release takes place.

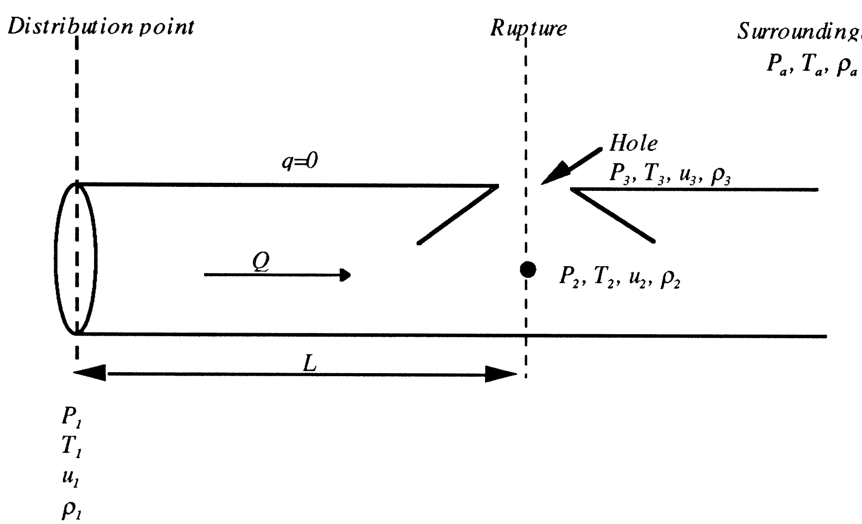

Fig. 1. Accident scenario.

The conditions at the various points are considered:
Point 1, at the beginning of the pipe
Point 2, inside the pipe and on a level with the hole
Point 3, at the hole
point in the surroundings, at atmospheric pressure.

The following hypothesis are assumed: (a) the model is devised for gas distribution systems operating at low and medium pressure, with relatively small pipe diameters (up to 350 mm i.d.) and lengths up to approximately 5 km. This means that ideal gas properties are assumed (with C_p = constant). The model can be applied then at pressures at which ideal gas behaviour exists; (b) the model does not take into account the initial depressurization (before the gas release) due to the pressure drop undergone by the gas while flowing through the pipe. In fact, this effect is small and, in any case, neglecting it implies a certain margin of safety; (c) isentropic flow at the release point and adiabatic flow in the pipe are assumed; (d) a model of flow essentially 1D is assumed.

The conditions downstream of the release point are relatively important only in the case of total rupture of the pipe. But even in this case, the contribution to the release of this downstream length of pipe will be small and will last a very short time (of the same order of magnitude as that corresponding to the unsteady state emptying of the pipe).

By applying the energy and momentum equations to the adiabatic flow through a pipe, the following equation is obtained:

$$\frac{k+1}{k}\ln\left(\frac{P_1 T_2}{P_2 T_1}\right) + \frac{M}{RG^2}\left(\frac{P_2^2}{T_2} - \frac{P_1^2}{T_1}\right) + \left(\frac{4 f L_e}{D}\right) = 0 \tag{1}$$

where L_e is the equivalent length of the pipe, which can be calculated using the following expression:

$$L_e = L + \sum N_i \cdot K_i \cdot \left(\frac{D}{f}\right) \tag{2}$$

In this expression, the Fanning friction factor can be calculated as follows:

For Re < 100 000, with the classical Blasius equation:

$$f = 0.079\,\mathrm{Re}^{-0.25} \tag{3}$$

For Re > 100 000, the Colebrook equation can be used. In this case, for smooth polyethylene pipes, this equation has been transformed into an explicit expression [12]:

$$f = 0.0232\,\mathrm{Re}^{-0.1507} \tag{4}$$

The mass discharge rate at the orifice can be calculated using the following expression, obtained from the continuity equation and the law of ideal gases for an isentropic expansion:

$$Q_m = A_{or} \cdot P_2 \cdot \sqrt{\frac{2M}{R \cdot T_2}\,\frac{k}{k-1}\left[\left(\frac{P_a}{P_2}\right)^{\frac{2}{k}} - \left(\frac{P_a}{P_2}\right)^{\frac{k+1}{k}}\right]} \tag{5}$$

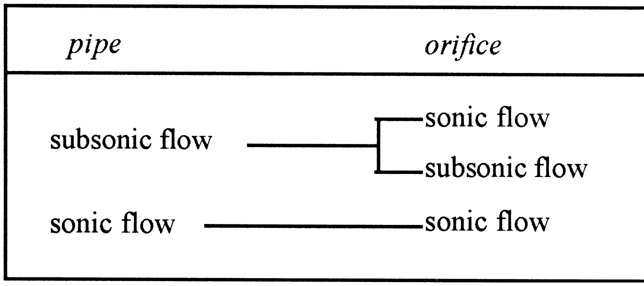

Fig. 2. Different possibilities of release.

An empirical discharge coefficient is often used in this expression; for situations of uncertainty (for example, when the exact shape of the hole is unknown), a conservative value of 1 is usually recommended; this value has been taken here.

The value of the flow-rate at the hole will depend on whether the flow is sonic or subsonic. This will be established by the critical pressure ratio (CPR), which indicates, for each case, the transition from sonic flow to subsonic flow:

$$\mathrm{CPR} = \frac{P_\mathrm{a}}{P_{2\mathrm{c}}} = \left(\frac{2}{k+1}\right)^{\frac{k}{k-1}} \tag{6}$$

$P_{2\mathrm{c}}$ is the critical pressure at Point 2, above which there will always be sonic flow at the outlet (release into the environment, i.e., towards atmospheric pressure). If the pressure inside the pipe becomes equal to or lower than $P_{2\mathrm{c}}$, the flow at the outlet will be subsonic. For the case of natural gas, with $k = 1.28$, and for a release towards atmospheric pressure, the value of the critical pressure is $P_{2\mathrm{c}} = 1.82$ bar abs.

The various possibilities are shown in Fig. 2. If there is subsonic flow in the pipe, the flow at the orifice can be either sonic or subsonic, depending on the ratio between the diameter of the pipe and that of the hole. If there is sonic flow in the pipe (at Point 2), there will always be sonic flow at the orifice. This leaves, therefore, three different situations to be analyzed.

3.1. Subsonic flow in the pipe, sonic flow at the hole

If it is assumed that all the gas flowing through the pipe also flows through the hole, the following mass balance can be established:

$$Q_m = A_{\mathrm{or}} \cdot u_3 \cdot \rho_3 = A_\mathrm{c} \cdot u_2 \cdot \rho_2 = A_\mathrm{c} \cdot u_1 \cdot \rho_1 \tag{7}$$

In this case, although the flow in the pipe is subsonic, the ratio between pipe diameter and hole diameter has a value such that gas release takes place with sonic velocity at the hole. For sonic flow at the hole, Q_m does not depend on P_a, and taking into account Eq. (6), Eq. (5) becomes:

$$Q_m = A_{\mathrm{or}} P_2 \sqrt{\frac{M}{RT_2} \cdot k \cdot \left(\frac{2}{k+1}\right)^{\frac{k+1}{k-1}}} \tag{8}$$

By applying the continuity law to the flow of gas through the pipe and the orifice, and expressing the mass flux as a function of Mach number, the following expression is obtained for the mass flux through the pipe and the hole:

$$G = \frac{A_{\mathrm{or}}}{A_{\mathrm{c}}} P_2 \sqrt{\frac{M}{RT_2} k \left[\frac{2}{k+1} \right]^{\frac{k+1}{k-1}}} = \mathrm{Ma}_1 P_1 \sqrt{\frac{kM}{RT_1}} = \mathrm{Ma}_2 P_2 \sqrt{\frac{kM}{RT_2}} \tag{9}$$

By introducing this value of G into Eq. (1), we obtain:

$$\frac{k+1}{k} \ln\left[\frac{P_1 T_2}{P_2 T_1} \right] + \frac{M}{R\left(\frac{A_{\mathrm{or}}}{A_{\mathrm{c}}} \right)^2 P_2^2 \frac{M}{RT_2} k \left(\frac{2}{k+1} \right)^{\frac{k+1}{k-1}}} \left[\frac{P_2^2}{T_2} - \frac{P_1^2}{T_1} \right] + \left[\frac{4fL_{\mathrm{e}}}{D} \right] = 0 \tag{10}$$

Taking into account the following relationships:

$$T_2 = \frac{Y_1}{Y_2} \cdot T_1 \tag{11}$$

$$P_2 = \frac{\mathrm{Ma}_1}{\mathrm{Ma}_2} \sqrt{\frac{Y_1}{Y_2}} \cdot P_1 \tag{12}$$

$$\rho_2 = \frac{\mathrm{Ma}_1}{\mathrm{Ma}_2} \sqrt{\frac{Y_2}{Y_1}} \cdot \rho_1 \tag{13}$$

where

$$Y_j = 1 + \left(\frac{k-1}{2} \right) \cdot \mathrm{Ma}_j^2 \tag{14}$$

and substituting them into Eq. (10), the following expression of the equation of energy is obtained which defines the present system:

$$\frac{k+1}{2k} \ln\left[\frac{\mathrm{Ma}_2^2}{\mathrm{Ma}_1^2} \frac{Y_1}{Y_2} \right] + \frac{\left(\frac{A_{\mathrm{c}}}{A_{\mathrm{or}}} \right)^2}{k \left[\frac{2}{k+1} \right]^{\frac{k+1}{k-1}}} \left(1 - \frac{\mathrm{Ma}_2^2}{\mathrm{Ma}_1^2} \right) + \frac{4fL_{\mathrm{e}}}{D} = 0 \tag{15}$$

In these conditions, the following relationships apply:

$$P_2 > P_1 \cdot \mathrm{Ma}_1 \sqrt{\frac{2Y_1}{k+1}} \tag{16}$$

$$\frac{P_{\mathrm{a}}}{P_2} < \mathrm{CPR} \tag{17}$$

The parameters corresponding to the point of release, Point 3, are defined for sonic flow as follows:

$$P_3 = \left(\frac{2}{k+1}\right)^{\frac{k}{k-1}} \cdot P_2 \tag{18}$$

$$T_3 = \left(\frac{2}{k+1}\right) \cdot T_2 \tag{19}$$

$$\rho_3 = \left(\frac{2}{k+1}\right)^{\frac{1}{k-1}} \cdot \rho_2 \tag{20}$$

and the parameters defining the gas at atmospheric pressure, for the same situation, are finally:

$$T_a = \left(\frac{P_a}{P_3}\right)^{\frac{k-1}{k}} \cdot T_3 \tag{21}$$

$$\rho_a = \left(\frac{P_a}{P_3}\right)^{\frac{1}{k}} \cdot \rho_3 \tag{22}$$

3.2. Subsonic flow in the pipe and at the orifice

In this case, the following relationship applies:

$$\frac{P_a}{P_2} \geq \text{CPR} \tag{23}$$

The mass discharge rate is now given by Eq. (5).

By applying the continuity equation to the flow of gas through the pipe and through the hole, and expressing the mass flux as a function of Mach number, the following expression is obtained:

$$G = \frac{A_{or}}{A_c} P_2 \sqrt{\frac{M}{RT_2} \frac{2k}{k-1} \left[\left(\frac{P_a}{P_2}\right)^{\frac{2}{k}} - \left(\frac{P_a}{P_2}\right)^{\frac{k+1}{k}} \right]} = \text{Ma}_1 P_1 \sqrt{\frac{kM}{RT_1}}$$

$$= \text{Ma}_2 P_2 \sqrt{\frac{kM}{RT_2}} \tag{24}$$

Taking into account again Eqs. (11)–(13), and introducing the value of G given by Eq. (24), the equation of energy can be expressed finally as:

$$\frac{k+1}{2k}\ln\left[\frac{\mathrm{Ma}_2^2}{\mathrm{Ma}_1^2}\frac{Y_1}{Y_2}\right]+\frac{k-1}{2k}\frac{\left(\dfrac{A_\mathrm{c}}{A_\mathrm{or}}\right)^2}{\left[\left(\dfrac{P_\mathrm{a}}{P_1}\dfrac{\mathrm{Ma}_2}{\mathrm{Ma}_1}\sqrt{\dfrac{Y_2}{Y_1}}\right)^{\frac{2}{k}}-\left(\dfrac{P_\mathrm{a}}{P_1}\dfrac{\mathrm{Ma}_2}{\mathrm{Ma}_1}\sqrt{\dfrac{Y_2}{Y_1}}\right)^{\frac{k+1}{k}}\right]}$$
$$\times\left(1-\frac{\mathrm{Ma}_2^2}{\mathrm{Ma}_1^2}\right)+\frac{4fL_\mathrm{e}}{D}=0 \tag{25}$$

This expression, together with the equation of continuity (Eq. (5), describes the phenomenon for this case.

As the flow is subsonic, the conditions at Point 3 and the atmospheric ones have the same values, given by Eqs. (21) and (22) by replacing P_3, T_3 and ρ_3 with the values corresponding to the conditions at Point 2.

In the case of full bore hole, Point 2 and Point 3 are the same. Therefore, there is no isentropic expansion between these two points (i.e. $P_2 = P_3 = P_\mathrm{a}$); Eq. (5) cannot be used and the mass balance (Eq. (24)) becomes:

$$G=\mathrm{Ma}_1 P_1\sqrt{\frac{kM}{RT_1}}=\mathrm{Ma}_2 P_2\sqrt{\frac{kM}{RT_2}} \tag{24b}$$

This is the expression which must be used together with Eq. (1) and Eqs. (11)–(13) to solve this case.

3.3. Sonic flow in the pipe (at Point 2) and at the hole

When $P_2 < P_1 \cdot \mathrm{Ma}_1\sqrt{\frac{2Y_1}{k+1}}$ the flow in the pipe becomes sonic at Point 2. In this case, the flow through an orifice can only be sonic.

Now, the starting point is the expression which gives the mass discharge rate through an orifice (sonic flow):

$$Q_m=A_\mathrm{or}\cdot\mathrm{Ma}_1\cdot P_1\sqrt{\frac{M}{RT_1}\cdot k\cdot\left(\frac{2}{k+1}\right)^{\frac{k+1}{k-1}}} \tag{26}$$

Following the same path as in the previous cases, and applying the following relationships:

$$\frac{P_2^2}{P_1^2}=\mathrm{Ma}_1^2\cdot\frac{2Y_1}{k+1} \tag{27}$$

$$\frac{P_1^2 T_2}{P_2^2 T_1}=\frac{1}{\mathrm{Ma}_1^2} \tag{28}$$

$$\left[\frac{P_1 T_2}{P_2 T_1}\right]^2=\frac{2Y_1}{(k+1)\mathrm{Ma}_1^2} \tag{29}$$

the following equation is obtained for the energy balance:

$$\frac{k+1}{2k}\ln\left[\frac{2Y_1}{(k+1)\mathrm{Ma}_1^2}\right]+\frac{\left(\dfrac{A_c}{A_{or}}\right)^2}{k\left[\dfrac{2}{k+1}\right]^{\frac{k+1}{k-1}}}\left(1-\frac{1}{\mathrm{Ma}_1^2}\right)+\frac{4fL_e}{D}=0 \qquad (30)$$

The conditions at Point 2 are now given by the following relationships:

$$P_2=\mathrm{Ma}_1\sqrt{\frac{2Y_1}{k+1}}\cdot P_1 \qquad (31)$$

$$T_2=\frac{2Y_1}{k+1}\cdot T_1 \qquad (32)$$

$$\rho_2=\mathrm{Ma}_1\sqrt{\frac{k+1}{2Y_1}}\cdot \rho_1 \qquad (33)$$

And the parameters at Point 3 and for the atmospheric conditions are defined by Eqs. (18)–(22).

Eqs. (15), (25) and (30) must be solved using an iterative method. In this case, the secant method has been used. In the case of subsonic flow in the pipe and at the hole, the Wegstein method to accelerate convergence [13] has been used to solve faster the system formed by the equation of continuity and the equation of energy.

Table 1 shows the sets of equations to be solved for each case in steady state, when $Q_m < Q_{max}$.

4. Examples of application

To check the validity of the proposed model, it has been applied to two different accident scenarios (in the first one, together with the aforementioned 'hole' and 'pipe' models [5]). In the following paragraphs the results are presented and discussed.

Case A: Partial or complete breaking of a pipe in which natural gas is flowing, with the following conditions:

pressure inside the pipe (at the initial point), $P_1 = 5$ bar abs;
temperature of the gas inside the pipe, $T_1 = 288$ K;
i.d. of the pipe, $D = 163.6$ mm;
distance between the point at which the pipe is damaged and the initial point, $L = 1000$ m;
molecular weight of natural gas, $M = 17.4$ kg/kmol;
density of natural gas at Point 1, $\rho_1 = 3.68$ kg/m^3;
viscosity of natural gas, $\mu_1 = 1.01 \times 10^{-5}$ kg/ms;
fittings: it is supposed that there is a 90° T ($K = 0.5$) every 75 m (i.e. 13 T's);
it is supposed that the regulator located at the feeding point (1) has no limit for the flow-rate, P_1 thus always being constant.

Table 1
Set of equations to be solved in steady state and unsteady state

Case	Subsonic flow in the pipe, sonic flow at the hole[a]	Subsonic flow in the pipe, subsonic flow at the hole[b]	Sonic flow in the pipe (Point 2), sonic flow at the hole[a]
Steady state, without any limitation on the flow-rate or $Q_m < Q_{max}$	Unknown variables: u_1, u_2, P_2, T_2 equations: $\frac{A_{or}}{A_c} P_2 \sqrt{\frac{M}{RT_2} k \left[\frac{2}{k+1}\right]^{\frac{k+1}{k-1}}} = \mathrm{Ma}_2 P_2 \sqrt{\frac{kM}{RT_2}}$ (9a) $\mathrm{Ma}_1 P_1 \sqrt{\frac{kM}{RT_1}} = \mathrm{Ma}_2 P_2 \sqrt{\frac{kM}{RT_2}}$ (9b) Eq. (11) Eq. (15)	Unknown variables: u_1, u_2, P_2, T_2 Equations: Eq. (11) $\frac{A_{or}}{A_c} P_2 \sqrt{\frac{M}{RT_2} \frac{2k}{k-1} \left[\left(\frac{P_a}{P_2}\right)^{\frac{2}{k}} - \left(\frac{P_a}{P_2}\right)^{\frac{k+1}{k}} \right]} = \mathrm{Ma}_2 P_2 \sqrt{\frac{kM}{RT_2}}$ (24a) Eq. (24b) Eq. (25)	Unknown variables: u_1, Q_m Equations: Eq. (26) Eq. (30)
Steady state, with a limitation on the flow-rate ($Q_m \geq Q_{max}$)	Unknown variables: P_1, u_2, P_2, T_2 Equations: The same set as in the previous case	Unknown variables: P_1, u_2, P_2, T_2 Equations: The same set as in the previous case	Unknown variables: P_1, Q_m Equations: The same set as in the previous case
Unsteady state	Unknown variables: P_1, T_1, u_1, u_2, P_2, T_2 Equations: The same set as in the previous case plus Eqs. (40) and (41)	Unknown variables: P_1, T_1, u_1, u_2, P_2, T_2 Equations: The same set as in the previous case plus Eqs. (46) and (48)	Unknown variables: P_1, T_1, u_1, Q_m Equations: The same set as in the previous case plus Eqs. (40) and (41)

[a] In these cases, in unsteady state, Eq. (35) must be used to calculate t_c.
[b] In this case, for full bore hole P_2 is known ($P_2 = P_a$), Eq. (1) must be used instead of Eq. (25) and Eq. (24a) is not required.

The value of the release flow-rate, as calculated according to the three models, has been plotted as a function of the hole diameter in Fig. 3. As can be observed, for small values of hole diameter, the new model gives results very close to those obtained with the ‘hole model’; this is due to the existence of very small friction loss and a practically negligible expansion of the gas inside the pipe. However, for large diameter holes, the values given by this model approach those of the ‘pipe model’. In the case of complete breaking of the pipe (diameter of the hole = diameter of the pipe), Fig. 3 shows that the new model gives the same value than would be obtained from the ‘pipe model’.

Over the whole range of possible hole diameters, the new model gives results which lay between those of the ‘pipe’ and ‘hole’ models and which are much closer to the real values of release flow-rate, as it takes into consideration the change in the flow inside the pipe, and the variation in pressure and temperature. Figs. 4 and 5 show the prediction of the pressure and temperature of the gas at the point of release, as a function of hole diameter. The different trends existing for the zones corresponding to sonic and subsonic flow can be observed.

Case B: In practice, the feeding points of a natural gas distribution net are controlled by flow-rate regulation valves (see the Appendix), which impose a limit on the

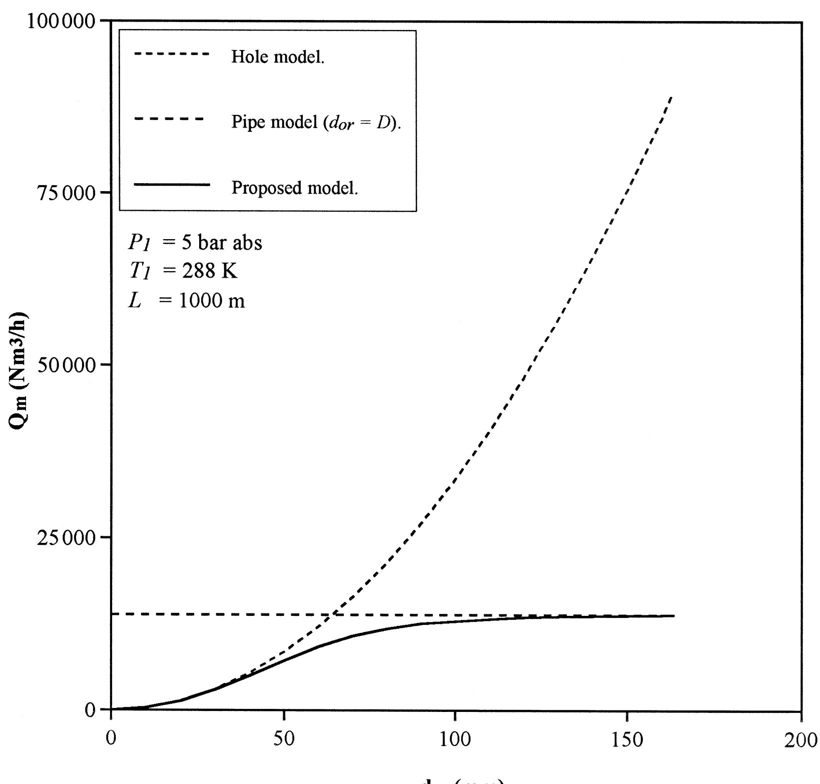

Fig. 3. Variation in release flow-rate as a function of hole diameter, according to hole model, pipe model and the new (hole–pipe) model.

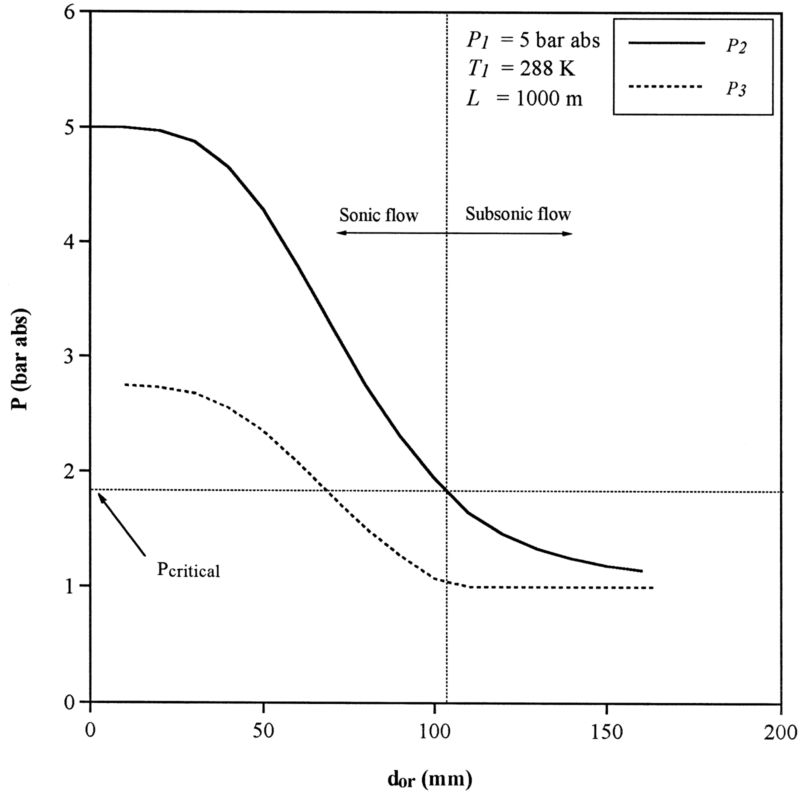

Fig. 4. Prediction of the pressure of the gas at the point of release, according to the pipe–hole model.

maximum possible flow-rate in the network. Therefore, the release flow-rate cannot be higher than this maximum value; this means that if the release flow-rate reaches this maximum value, the pressure at the feeding point will no longer be constant and the pressure throughout the pipe will decrease (although, as in the previous example, the situation will correspond to a steady-state condition). In this case ($Q_m \geq Q_{max}$), the set of equations to be solved is the same than for the previous case for all the possible situations, but with a change in the boundary conditions. Now the value of u_1 is known and the value of P_1 is unknown (Table 1).

The conditions will now be assumed to be the same as in Example 1, but with the flow-rate limited to a maximum value of 9130 Nm^3/h ($P_E = 9$ bar abs) by a Tartarini series FL-BP DN50 regulator.

Fig. 6 shows the variation in the release flow-rate as a function of the diameter of the hole; the flow-rate reaches a maximum value of 9130 Nm^3/h at a d_{or} of approximately 65 mm. This implies that from this value of d_{or} the pressure at Point 1 (the feeding point) will decrease if the hole diameter is larger (Fig. 7). It can be observed also in Fig. 7 that the limitation in the flow-rate gives rise to a change in the flow at the hole (from sonic to subsonic). This change occurs at a hole diameter smaller than that correspond-

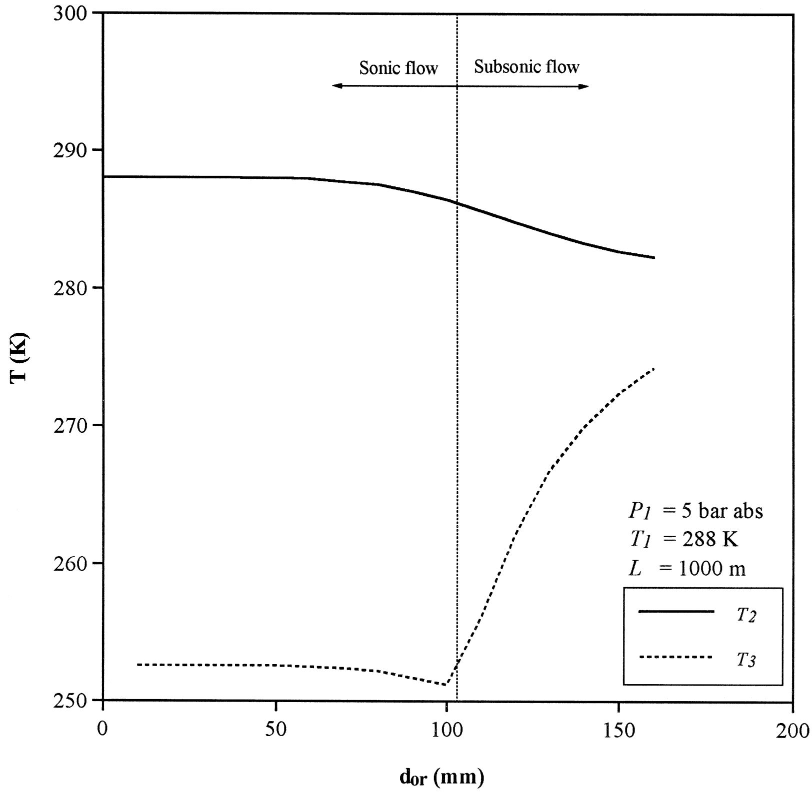

Fig. 5. Prediction of the temperature of the gas at the point of release, according to the pipe–hole model.

ing to the case in which the flow-rate has not reached the limiting value or there is no regulator. This is why the temperature at Point 3 and T_a reach the same value at a smaller hole diameter (Fig. 8); these two temperatures and the temperature at Point 2 have also the same value at a smaller hole diameter. Fig. 8 shows also that at large hole diameters (but smaller than pipe diameter) these three temperatures have already the same value.

The variation of gas density at the different points as a function of the hole diameter has been plotted in Fig. 9. As can be observed, for large (but smaller than pipe diameter) holes the densities at Point 2 and Point 3 are the same than that corresponding at atmospheric conditions.

5. Unsteady state

The above analysis is related to a stationary state, i.e. the release flow-rate has a constant value as a function of time.

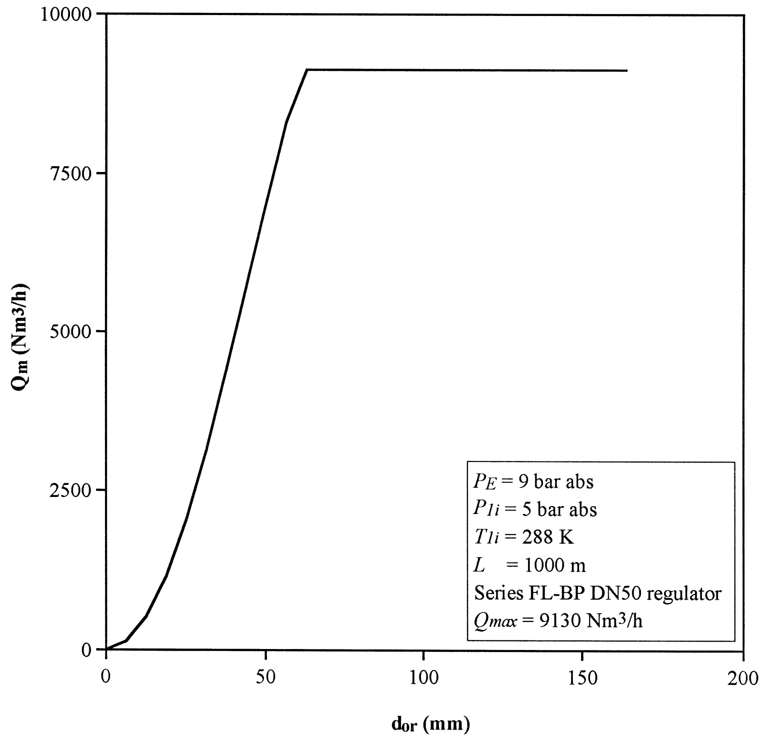

Fig. 6. Variation in the release flow-rate as a function of hole diameter, with a limiting regulator.

There will be an initial transient phase following the rupture. Nevertheless, for the situations (pipe length and diameter) for which the model has been developed, this step will last usually a few seconds or even less than a second; this time can be considered negligible as compared to the time during which the release takes place.

However, in practice, after a certain time from the beginning of the release, the feed of gas into the system will be stopped automatically by means of a regulator (as a reaction to an excessively high flow-rate) or by hand. From that moment on, the release flow-rate will start to decrease up to the end of the emergency.

Unsteady state has been analyzed by Flatt [10,14] and Olorunmaiye and Imide [11] for the case of long, high-pressure gas pipelines. However, we have not found any publications concerning medium and low-pressure gas distribution systems.

This unsteady state period is interesting to analyse inasmuch as it shows how long the release will last after the gas feed is closed, as well as the overall amount of gas released. In the following paragraphs this situation is modelled.

Once the regulator is closed because of the emergency, there is a certain volume of gas in the pipe; therefore, the following continuity balance can be established as a function of time:

$$Q_m(t) = -V_P \cdot \frac{d\rho(t)}{dt} \tag{34}$$

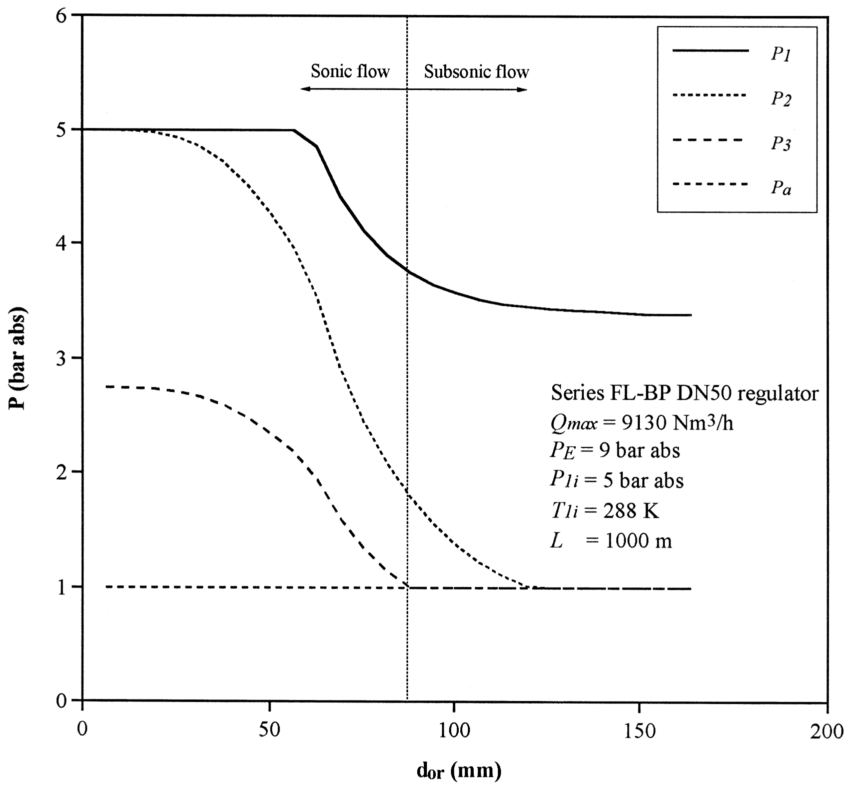

Fig. 7. Variation in gas pressure as a function of hole diameter, with a limiting regulator.

In this expression, ρ is an average value obtained from the integration of $\mathrm{d}\rho/\mathrm{d}L$ over all the length of the pipe.

Even if initially the release flow was sonic, once the gas feed is closed the pressure at the feeding point (and throughout the pipe) will start to decrease as a function of time and after a certain period the release will become subsonic. The transition from sonic to subsonic flow will take place when $P_{\mathrm{a}}/P = \mathrm{CPR}$; the time elapsed from the closing of the feed until that moment is called the critical time (t_{c}) and can be calculated with the following equation [3]:

$$t_{\mathrm{c}} = \frac{1}{\alpha}\left[\frac{1}{\left(\frac{k+1}{2}\right)^{1/2}\left(\frac{P_{\mathrm{a}}}{P_0}\right)^{(k-1)/2k}} - 1\right] \tag{35}$$

where

$$\alpha = \frac{Q_{m_0}\cdot(k-1)}{2\cdot m_0} \tag{36}$$

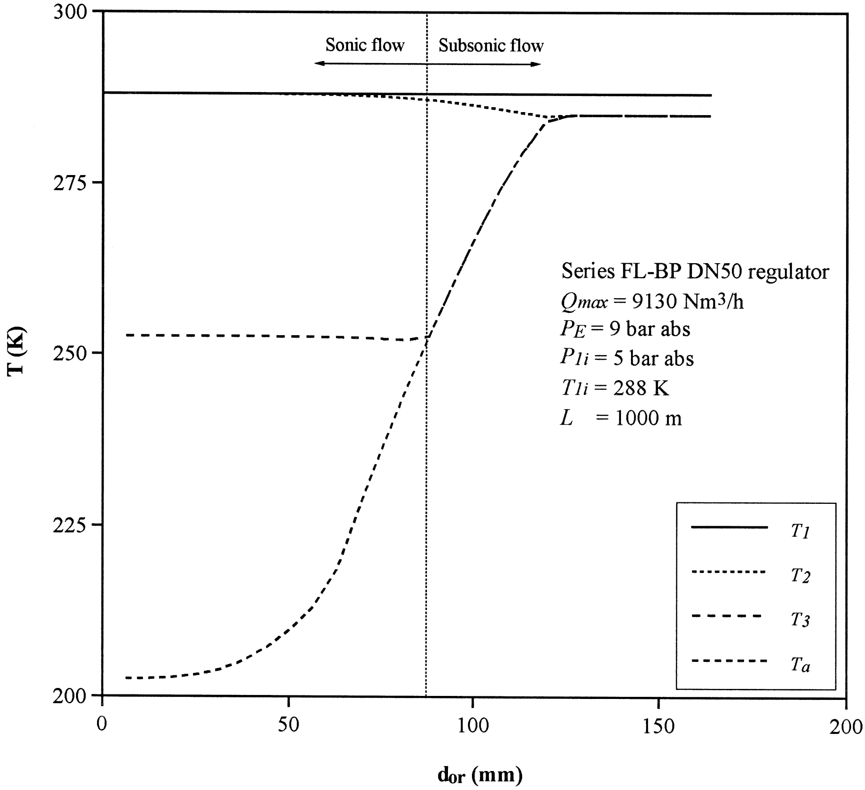

Fig. 8. Variation in gas temperature as a function of hole diameter, with a limiting regulator.

By substituting into Eq. (34) the expression for the release flow-rate (Eq. (26) for sonic flow, or Eq. (5) for subsonic flow), the following expressions are obtained for sonic flow and subsonic flow, respectively,

$$t = -V_{\mathrm{P}} \int_{\rho_0}^{\rho} \frac{\mathrm{d}\rho(t)}{A_{\mathrm{or}} \cdot \left[k \cdot P(t) \cdot \rho(t) \cdot \left(\dfrac{2}{k=1} \right)^{(k+1)/(k-1)} \right]^{1/2}} \tag{37}$$

$$t - t_{\mathrm{c}} = -V_{\mathrm{P}} \int_{\rho_0}^{\rho} \frac{\mathrm{d}\rho(t)}{A_{\mathrm{or}} \cdot \left[P(t) \cdot \rho(t) \cdot \left(\dfrac{2k}{k+1} \right) \cdot \beta \right]^{1/2}} \tag{38}$$

where

$$\beta = \left(\frac{P_{\mathrm{a}}}{P(t)} \right)^{2/k} \cdot \left[1 - \left(\frac{P_{\mathrm{a}}}{P(t)} \right)^{(k-1)/k} \right] \tag{39}$$

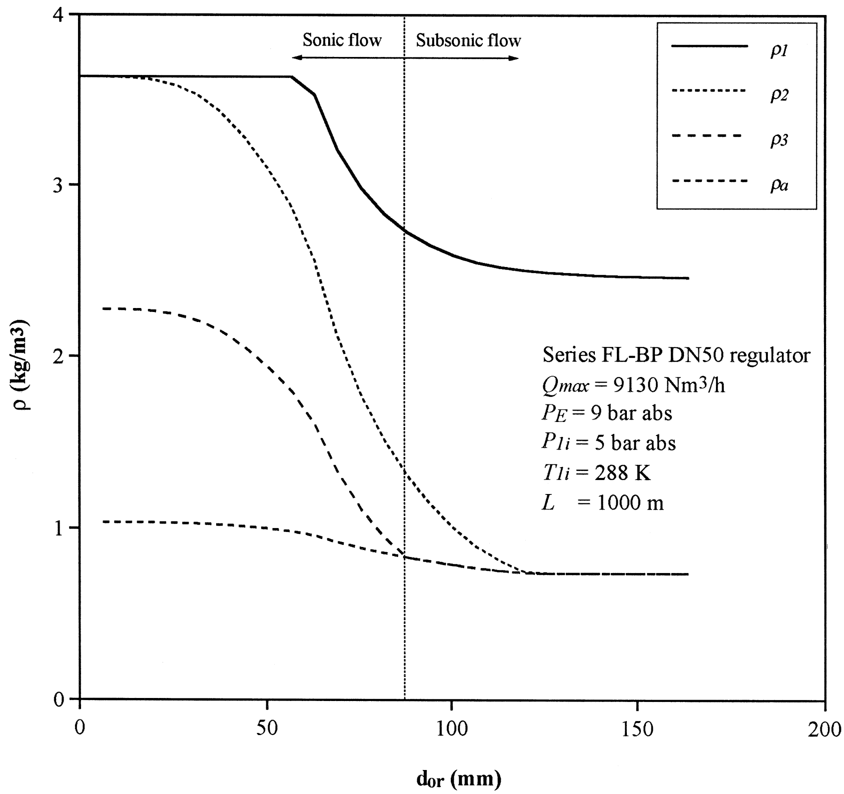

Fig. 9. Variation in gas density as a function of hole diameter, with a limiting regulator.

Assuming that the variation as a function of time in the parameters which characterize the gas in the pipe (P, T, ρ) corresponds to an isentropic expansion, the integration of Eq. (37) leads to the following results:

$$\frac{P(t)}{P_0} = [F(t)]^{2k/(k-1)} \tag{40}$$

$$\frac{T(t)}{T_0} = [F(t)]^2 \tag{41}$$

$$\frac{\rho(t)}{\rho_0} = [F(t)]^{2/(k-1)} \tag{42}$$

$$Q_m(t) = Q_{m_0} \cdot [F(t)]^{(k+1)/(k-1)} \tag{43}$$

$$m(t) = m_0 \cdot \left[1 - F(t)^{2/(k-1)}\right] \tag{44}$$

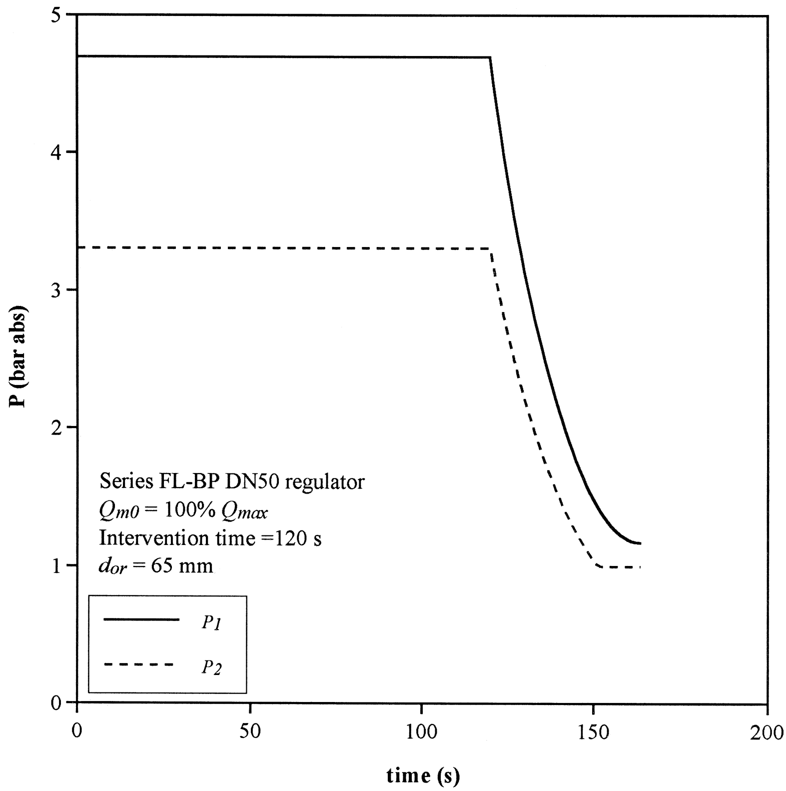

Fig. 10. Evolution of pressure at Points 1 and 2 as a function of time, for unsteady state.

where

$$F(t) = \frac{1}{1 + \alpha \cdot t} \tag{45}$$

This set of equations, taken from Woodward and Mudan [3], defines the release model for unsteady state in the case of sonic flow.

In the case of subsonic flow, it is not possible to integrate Eq. (38) analytically. Therefore, the equation corresponding to this case can be left in differential form:

$$\frac{d\Phi}{dt} = -\frac{[\Phi - B]^{1/2}}{A' \cdot \Phi^{C'}} \tag{46}$$

where

$$\Phi = \frac{T(t)}{T_{0_{\rm sub}}};\ B = \left(\frac{P_{\rm a}}{P_{0_{\rm sub}}}\right)^{(k-1)/k};\ C' = \frac{2-k}{k-1};\ A' = \frac{m_{0_{\rm sub}}}{(k-1)Q_{m_{0\rm sub}}}(1-B)^{1/2} \tag{47}$$

and

$$\frac{P(t)}{P_{0_{sub}}} = \left[\frac{T(t)}{T_{0_{sub}}}\right]^{\frac{k}{k-1}} \tag{48}$$

Eq. (46) can be solved numerically using an ODE method (for example, fourth order Runge–Kutta method).

These relationships defining the model in unsteady state (from Eqs. (40)–(44) for sonic flow and from Eqs. (46)–(48) for subsonic flow), together with the equations of momentum, energy and continuity—developed in Section 3 for each case—give a set of equations which allows the calculation of each variable of the system as a function of time (Table 1).

In the application of these equations it must be taken into account that the flow regime at the hole can change during the release, varying from sonic to subsonic flow.

In order to study the evolution of the different release parameters as a function of time, the model will be applied in the following paragraphs to an accident scenario.

Case C: The situation corresponds to that of Case B, but with a hole diameter of 65 mm and an intervention time (closing of the regulator) of 2 min.

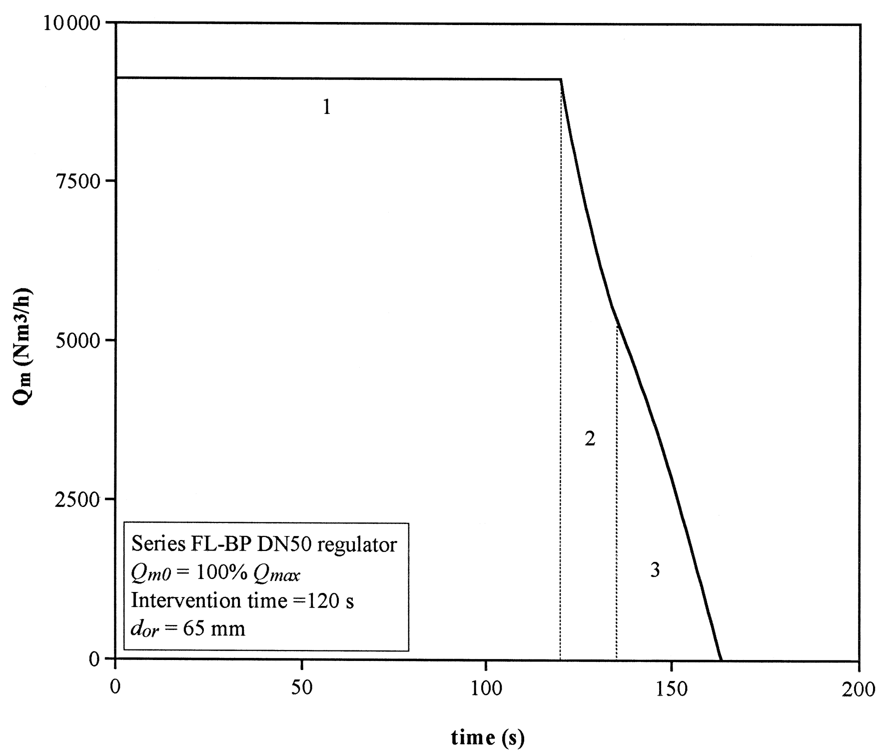

Fig. 11. Variation of release flow-rate with time for unsteady state.

The evolution of both pressures (P_1 and P_2) as a function of time, according to the new model, can be seen in Fig. 10. After the first 2 min with a constant value, both pressures start to decrease when the regulator is closed, tending towards atmospheric pressure. It should be observed that after the moment at which pressure at Point 2 reaches the atmospheric value, there is still a certain release flow-rate due to the pressure gradient inside the pipe.

The variation in the flow-rate as a function of time can be seen in Fig. 11. The existence of three zones can be observed. The first one (for this case, the first 2 min) corresponds to the steady state, with maximum release flow-rate. The second one, ranging from $t = 120$ s up to approximately $t = 135$ s, corresponds to a situation in which the feed of gas has already been stopped, but the flow through the hole is still sonic. Finally, there is a third zone, ranging from the instant $t = 135$ s up to the moment at which the flow-rate is practically zero ($t = 164$ s), which corresponds to subsonic flow through the hole. Of course, the duration of these different situations will depend on the circumstances of the accident situation.

Finally, Fig. 12 gives the variation as a function of time in the amount of gas released through the hole and contained inside the pipe, respectively. As can be observed, once the release is finished, there is still a mass of gas contained in the pipe, which corresponds to the volume of the pipe filled by gas at the final pressure of 1 bar.

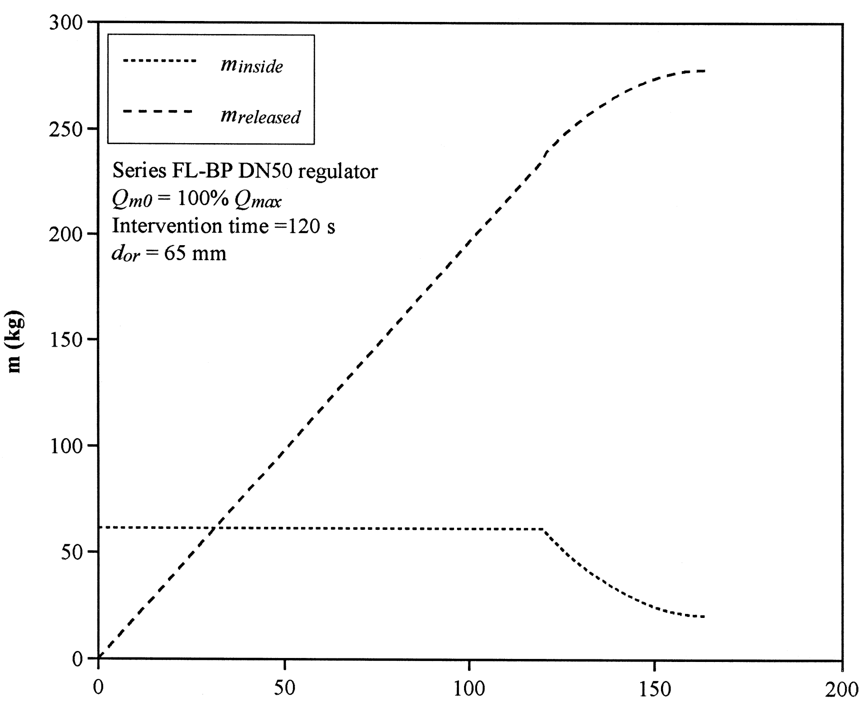

Fig. 12. Variation in the masses of gas released and inside the pipe, respectively, as a function of time.

6. Conclusions

The new model has been developed from the application of fundamental equations of fluid mechanics. The resulting equations (summarized in Table 1) can therefore be applied to many different situations concerning the accidental release of a gas through a damaged pipe in a gas distribution system (the model is not intended to be applied to large pipelines).

As a consequence, it has significant advantages over the models previously published and available: firstly, it covers a range of accident scenarios which could not be dealt with by the classical ‘pipe’ or ‘hole’, i.e. that ranging between a small hole and the complete breaking of the pipe. Secondly, it allows the analysis of the unsteady state which appears when a safety device or a regulator are closed, both for sonic and subsonic flow. Furthermore, the model takes into account the limitation on the flow-rate imposed by the existence of a flow regulator; in this case, the model also allows the calculation of the decrease in pressure over the pipe.

This model is therefore a step forward in the effort to develop more precise and powerful calculation tools to foresee the effects and consequences of potential accidents caused by the loss of containment of hazardous materials.

Acknowledgements

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Appendix

The regulator used in this work is the Tartarini series FL-BP DN50 regulator. The value of Q_{max} (in m^3/h at standard conditions) is described by the following equations.

For $\frac{P_E}{P_1} < 2$:

$$Q_{max} = 1118 \cdot P_E \cdot \sin\left(2.259 \cdot \sqrt{\frac{P_E - P_1}{P_E}}\right) \quad (A.1)$$

For $\frac{P_E}{P_1} \geq 2$:

$$Q_{max} = 1118 \cdot P_E \quad (A.2)$$

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